



Second Derivative General Linear Methods for Transient Analysis: A Robust Approach to Damped RLC Circuits Across Damping Conditions

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Abstract

Transient analysis of an electrical circuit is an important study in determining how electric circuits respond dynamically due to sudden changes, such as switching operations. The parallel RLC circuit has become crucial, both in academics and engineering fields, pertaining to designing various engineering devices: oscillators, filters, and dissipating energy that might be harmful to people or electronics. This paper examines the application of Second Derivative General Linear Methods (SGLMs) for solving the governing second-order ordinary differential equation of a damped parallel RLC circuit. SGLMs with second derivative have given with superior stability and accuracy for non-stiff and stiff systems. This study compared the given methods with other traditional methods under the conditions: as overdamped, critically damped, and underdamped. Results will find wide implications in design and optimization studies of electrical systems by providing a robust framework for accurate, efficient transient analysis. The results show that SGLMs achieved significantly lower absolute errors compared to classical methods. Specifically, the maximum error across all simulations was over 10 times smaller than that of RK4 (Runge-Kutta 4), and the Euler method exhibited even greater deviations. SGLMs remained stable even at larger step sizes (up to $h = 0.1$) where the other methods either became unstable or lost accuracy.

Keywords

SGLMs, Runge-Kutta 4, RLC Circuits, Ordinary Differential Equation

I. INTRODUCTION

Transient behavior analysis in electrical circuits is a fundamental tool for describing the performance and stability of systems under various conditions. From the simplest circuitry related to electrical engineering comes the RLC circuit, comprising of a resistor, and an inductor and capacitor, respectively. Such circuits find several uses in filtering and tuning applications, oscillation devices, and energy storage. With the application of sudden changes, such as step inputs, RLC circuits receive important insights into their dynamic when a transient response is applied [1]-[4]. Parallel RLC circuits are of special interest because they have a unique capability for handling energy losses and oscillatory behaviors. The dynamic response of these kinds of circuits is described by a second-order ODE (Ordinary Differential Equation) whose behavior depends on the damping factor.

Depending on the value of the damping coefficient, it is possible to get an overdamped, critically damped, or underdamped response of the system, with each type of response having different implications on the performance of the circuit. Traditional analysis methods for such circuits always have analytical solutions or numerical approximations like Euler's method and Runge-Kutta techniques. While these methods are effective in simple systems, most of them have problems regarding their accuracy and stability when applied to stiff systems or those with high-frequency constituents [5][6].

Recent advances in numerical methods have overcome some deficiencies in traditional approaches and equipped science with a new armory. Among these, such a development came forward the SGLMs. SGLMs used the second derivative with their process explicitly, however, the most methods do not use it to arrive their solutions This



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enhanced ability allows a wider range of application when dealing with problems that require much greater stability and accuracy. this approach performs effectively for both stiff and non-stiff systems, and it offers a flexible and reliable way to handle the complexities of solving ordinary differential equations, especially when step sizes need to change [7]-[9].

The SGLMs are used in this paper to solve some transient analyses of parallel RLC circuits under different damping conditions. The main goal is to illustrate the effectiveness of SGLMs in obtaining a more accurate and efficient solution to the governing second-order ODE of the circuit. A comparison of the performance of SGLMs with conventional numerical methods will reveal the advantage of the inclusion of second derivatives, particularly when stability and accuracy are important.

The rest of this paper is organized as follows. Section II presents an overview of SGLMs. Section III presents the dynamics of parallel RLC circuits and the mathematical framework of the SGLMs method. Then, section IV describes the methodology for applying this method to analyze the transient response of such circuits. Section V presents the results of the performed simulations, comparing the SGLMs method with conventional methods under different damping conditions. Finally, Section VI provides conclusions and suggestions for future work.

II. OVERVIEW OF SGLMS

The SGLMs form a new class of numerical methods for solving the ODE-IVPs $y'' = f(t, y, y')$. In contrast with most traditional methods, which obtain information from function values and first-order derivatives only, the SGLMs explicitly involve the second-order derivatives [8]. Therefore, they give better stability and higher accuracy in approaching the exact solution. Thus, SGLMs are very suitable to deal with both stiff and non-stiff differential equations; including systems with high-frequency components [10]. Second-order derivatives extend the framework of the GLM in the SGLM. An ODE system of the following form:

$$\frac{dy}{dt} = f(t, y), \quad \frac{d^2y}{dt^2} = \frac{d}{dt}f(t, y) = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial y}\right) f(t, y) \quad (1)$$

At each time step n , the SGLM updates the solution by combining the values of the previous solution with the first derivative $f(t, y)$ and the second derivative $\frac{d}{dt}f(t, y)$. The procedure is articulated as:

$$\begin{aligned} Y^{[n]} &= h(A \otimes I)f(Y^{[n]}) + h^2(A^* \otimes I)\bar{f}(Y^{[n]}) + (U \otimes I)y^{[n-1]} \\ y^{[n]} &= h(B \otimes I)f(Y^{[n]}) + h^2(B^* \otimes I)\bar{f}(Y^{[n]}) + (V \otimes I)y^{[n-1]} \end{aligned} \quad (2)$$

where $y^{[n]}$ is the approximate solution at the n^{th} time step, h is the step size, and $Y^{[n]}$ are stage values for intermediate calculations. The matrices A, A^*, B, B^*, U, V are coefficient matrices intended to enhance stability and precision. The expressions $f(Y)$ and $\bar{f}(Y)$ denote the first and second derivative values at the specified positions, respectively. SGLMs, by using second derivatives, manage rapidly changing dynamics and stiff equations more

proficiently than conventional approaches such as Runge-Kutta or Adams-Bashforth.

According to [11], SGLMs are designed to possess properties that guarantee they are robust and efficient. They achieve high stability—an important feature of stiff ODEs, for which other methods often show poor results unless one makes potentially harmful decisions regarding extremely small step-size values. Consistency and convergence are guaranteed in the satisfaction of order conditions relative to both first and second derivatives, making the global error decreasing as the step size approaches zero. Besides, SGLMs are very flexible because their coefficient matrices can be devised in such a way as to optimize their performance for specific types of ODE. Their efficiency comes due to the reduced number of steps required to arrive at given accuracy due to the involvement of second derivatives. Some of the very wide-ranging applications of SGLMs in many scientific and engineering disciplines include transient response problems arising in electrical circuits that have high-frequency components, mechanical system models with fast oscillations, a wide variety of analyses in fluid dynamics containing stiff nonlinearities, and stability assessments in control systems. On the other hand, some of the limiting factors for SGLMs are that detailed knowledge of the second derivatives is needed, which one might or might not possess. Extra coefficient matrix preconditioning may also be necessary in the case of stiff systems. Complex dynamics can add computational overhead in the computation of second derivatives [12][13]. Therefore, Second Derivative General Linear Methods signify a notable progression in numerical analysis, providing enhanced stability, precision, and versatility. Despite their complexity presenting obstacles, their benefits render them an effective instrument for addressing ordinary differential equations, especially in systems characterized by stiffness or high-frequency dynamics.

III. PARALLEL RLC CIRCUIT DYNAMICS

Differential equations are often used in modeling the electrical circuit dynamics. The simplest type of circuitry to handle transient phenomena is the parallel RLC circuit, which consists of a resistor (R), an inductor (L), and a capacitor (C), connected in parallel. This is what is called transient response, which occurs when the circuit is subjected to sudden changes in input; for example, simultaneous switching on a DC voltage source. For complete understanding of electrical system performance under steady-state and unsteady-state conditions, a good grasp of this behavior is vital [14]. Ref. [15] explains the application of Kirchhoff's Current Law (KCL) at a node state that the sum of currents at a node is equal to the total current flowing into a node. The application of this law will allow us to derive the governing equation of the parallel RLC circuit. A second-order linear differential equation is produced as a result of differentiating the complete equation with regard to time. This equation is responsible for regulating the voltage across the circuit.

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0 \quad (3)$$

The coefficient of the first derivative is denoted by the term $\frac{1}{RC}$, while the reciprocal of the natural frequency of oscillation cubed is denoted by the term $\frac{1}{LC}$. Whether the system shows oscillatory or non-oscillatory transient responses is determined by the damping factor (ζ), which is the factor that affects the behavior of the parallel RLC circuit. The following equation can be used to calculate the damping factor:

$$\zeta = \frac{1}{2R} \sqrt{\frac{L}{C}} \quad (4)$$

Based on the value of ζ , the behavior of the system can be divided into three distinct categories [5]:

- A. **Overdamped** ($\zeta > 1$): If there are no oscillations, the transitory response will decrease exponentially. In the event that the resistance R is large in comparison to the inductance L and the capacitance C , this phenomenon takes place.
- B. **Critically Damped** ($\zeta = 1$): The system is able to recover to a steady state in the lowest amount of time possible without oscillating. A perfect equilibrium between speed and stability is represented by this condition.
- C. **Underdamped** ($\zeta < 1$): The oscillations that make up the transient response have an amplitude that decreases exponentially. The resistance R is relatively low, which causes this to occur.

The formula that describes the resonant frequency ω_0 of the parallel RLC circuit is as follows:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (5)$$

This frequency is the natural oscillation rate of the system when dampening is not present so that it may be measured. It is also possible for the time constants associated with the inductor and capacitor to have an effect on the transient responsiveness of the circuit:

$$\tau_L = \frac{L}{R}, \quad \tau_C = RC \quad (6)$$

The time scale over which energy is lost in the circuit is being defined by these time constants. Parallel RLC circuits are used extensively in engineering and electronics. Common applications include [16]:

- A. **Filters**: It is possible to choose or suppress particular frequencies by employing parallel RLC circuits in band-pass and band-stop filters.
- B. **Oscillators**: As a result of the circuit's inherent oscillating characteristic, it is well-suited for the generation of sinusoidal signals inside of communication systems.
- C. **Energy Storage and Dissipation**: While the resistor is responsible for dissipating energy, the capacitor and inductor are responsible for storing energy. Because of this, these circuits are essential in power electronics and signal processing.

Altogether, the parallel RLC circuit represents an elementary but effective basis for studying transient phenomena in electrical systems. Oscillation, exponential decay, and resonance are some of the basic dynamics captured by the second-order differential equation describing the circuit. It would be possible to predict the response of the circuit to transient inputs based on a careful analysis of damping factor conditions and the system parameters. Hence, its vital status as a foundational model in electrical engineering.

IV. NUMERICAL METHODOLOGY

An ordinary differential equation of the second order is derived from Kirchhoff's Current Law, and it is used to model the system in order to assess the transient response of a parallel RLC circuit. Second Derivative General Linear Methods, which make use of second derivatives to achieve improved accuracy and stability, are the numerical methods that we employ in order to solve this ODE. A first-order system is initially reformulated from the second-order ordinary differential equation:

$$y_1 = v, \quad y_2 = \frac{dv}{dt}, \quad \frac{dy_1}{dt} = y_2, \quad \frac{dy_2}{dt} = -\frac{1}{RC}y_2 - \frac{1}{LC}y_1$$

The explanations of the first and second derivatives for the RLC circuit are:

$$f(Y) = \begin{bmatrix} y_2 \\ -\frac{1}{RC}y_2 - \frac{1}{LC}y_1 \end{bmatrix}, \quad \bar{f}(Y) = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \quad (7)$$

Where $f_1 = y_2$ and $f_2 = -\frac{1}{RC}y_2 - \frac{1}{LC}y_1$

We update the voltage y_1 and its derivative at each time step n as follows:

$$y_1^{[n+1]} = y_1^{[n]} + h \sum_{i=1}^s b_i f_1^{(i)} + h^2 \sum_{i=1}^s b_i' \bar{f}_1^{(i)}$$

$$y_2^{[n+1]} = y_2^{[n]} + h \sum_{i=1}^s b_i f_2^{(i)} + h^2 \sum_{i=1}^s b_i' \bar{f}_2^{(i)} \quad (8)$$

The values of the intermediate stage are calculated repeatedly as follows:

$$Y_i = y^{[n]} + h \sum_{j=1}^s a_{ij} f(Y_j) + h^2 \sum_{j=1}^s a_{ij}' \bar{f}(Y_j) \quad (9)$$

where a_{ij} and a_{ij}' denotes the coefficients for the intermediate stages and the number of stages in the method considered by s . The circuit parameters for the simulations were set at $L = 1H$ and $C = 0.25F$ and the resistance R was changed to examine three damping conditions:

- Underdamped: $R > \frac{1}{2} \sqrt{\frac{L}{C}}$
- Critically damped: $R = \frac{1}{2} \sqrt{\frac{L}{C}}$
- Overdamped: $R < \frac{1}{2} \sqrt{\frac{L}{C}}$

As a result of modifying the circuit parameters (R, L and C), the simulations investigate three different damping scenarios:

overdamped, critically damped, and an underdamped system. The following are important particulars:

- Time interval: $t \in [0, 1]$ seconds,
- Step sizes: $h \in [0.01, 0.1]$,
- Initial conditions: $v(0) = 1\text{ V}$ and $\frac{dv}{dt}(0) = 0$

V. NUMERICAL RESULTS

Numerical results that show the performance of SGLMs in solving the transient response of a damped parallel RLC circuit are compared with some traditional numerical methods like Euler and RK4 of fourth order. Besides, results under three damping conditions-namely, overdamping, critical damping, and underdamping-are obtained. Fig.1 compares the computed transient voltage response using Euler, RK4, and SGLMs against the exact analytical solution. Major differences in accuracy and stability can be seen for these methods. Specifically, Euler significantly deviates due to the first-order approximation from the exact solution, especially at smaller time steps, while RK4 obtains much better accuracy with minor discrepancies near the start of the response. Meanwhile, SGLMs remain closer to the exact solution during the simulation run and therefore give the most accurate results. Explicit inclusion of second derivatives in the method may be attributed to the increased stability and precision of the method. Stability of SGLMs is apparent also from Fig.1. To complement the numerical simulations and validate the stability advantage of SGLMs, we performed a stability analysis using the Boundary Locus Plot. The boundary locus method involves applying the SGLM to the standard test equation $\frac{dy}{dt} = \lambda y$, where $\lambda \in C$ and plotting the values of $h\lambda$ that yield bounded solutions.

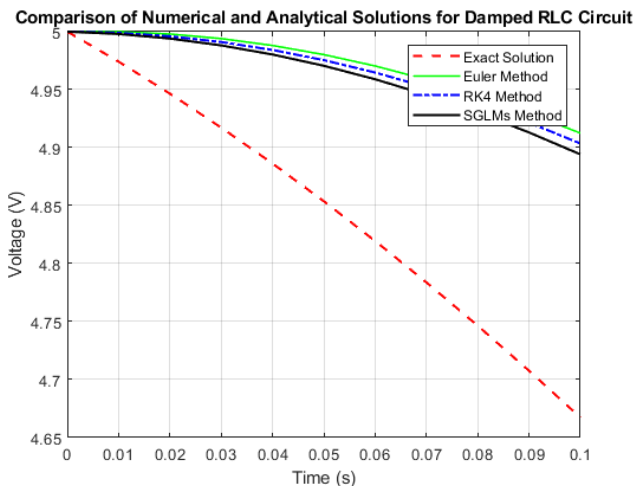


Fig.1. Comparison of transient voltage responses obtained using Euler, RK4, and SGLMs

Fig.2 shows the stability regions of the SGLM, RK4, and Euler methods in the complex plane. As evident, the stability region of SGLMs encompasses a much larger part of the left-half complex plane, making it well-suited for stiff systems with large negative eigenvalues. In contrast, the Euler method has a very limited stability region, and RK4, though better, does not fully cover the stiff regimes. Euler method cannot afford the stability issue, and it requires smaller time-step size to keep the error in an acceptable level. RK4 achieves higher stability but involves extra computational cost. SGLMs retain

stability even for a larger time step and hence would be well suited especially for stiff and rapidly decaying systems such as the damped RLC circuit. A minimum deviation from the exact solution shows the robustness of SGLMs for transient circuit analysis.

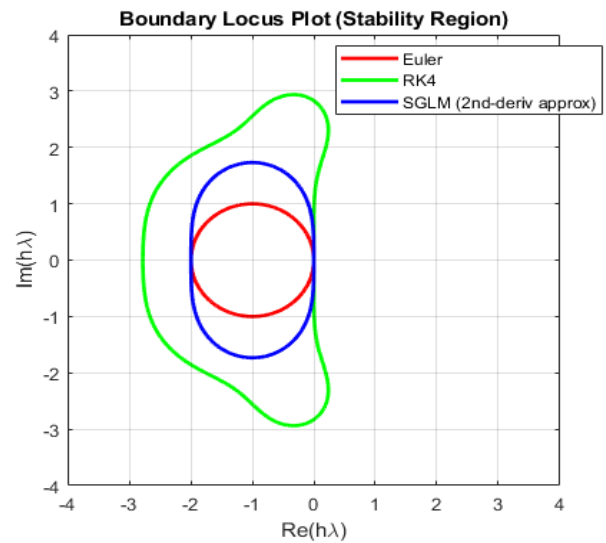


Fig.2. Stability regions of Euler, RK4, and SGLMs in the complex plane, illustrating the extended stability domain of SGLMs for stiff transient problems.

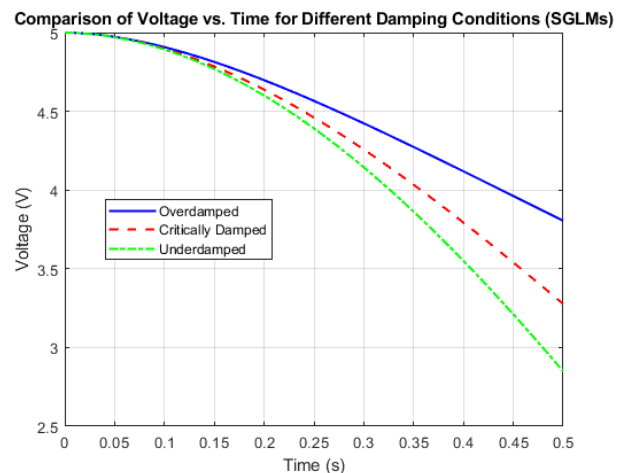


Fig.3. Transient voltage responses under overdamped, critically damped, and underdamped conditions using SGLMs, compared with the exact analytical solution.

To further evaluate the appropriateness of SGLMs, the transient response for three different damping cases: overdamped, critically damped, and underdamped were investigated. Fig.3, depicts voltage decay for each damping case obtained from the SGLM solution compared with the exact analytical solution. The overdamped circuit in this case decays exponentially in a slow fashion without oscillations. The solution given by SGLM lies close to the exact solution, catching the feature of a slowly decaying response characteristic of an overdamped system. In the case of critically damped circuit, the fastest possible return to steady state occurs without overshooting or oscillations. SGLM does catch this with very minimal error throughout the simulation.

Fig.3 demonstrate that the distinct behaviors of overdamped, critically damped, and underdamped systems are accurately represented. The slow exponential decay for

the overdamped condition, the fast and oscillation-free decay for the critically damped condition, and the oscillatory decay for the underdamped condition are all faithfully represented throughout the figures.

This comparison underlines the robustness of SGLMs, which keeps the accuracy and stability in a wide range of damping scenarios. Furthermore, it is confirmed from error analysis that the SGLM develops a negligible error concerning the exact solution within all damping conditions. The maximum error for SGLMs is at least one order of magnitude lower than in the RK4 and Euler methods.

Besides imaging quality and reconstruction stability, computational efficiency was also evaluated. While SGLMs require the quantities of second derivatives to be evaluated, more stable methods can use larger step sizes that in turn will give fewer total integration steps. As a result, the total computational cost becomes equal to that of RK4 and can be lower in stiff cases since the latter often demands smaller step sizes for stability.

The processing cost was briefly taken into account in addition to accuracy and stability. Despite requiring the evaluation of second derivatives, SGLMs' improved stability permits bigger time step sizes, which lowers the overall number of integration steps needed. SGLMs are therefore an effective option for transient RLC circuit simulations since the overall computational cost becomes comparable to, and in stiff situations possibly lower than, that of classical approaches like RK4.

The numerical results are analyzed for the simulated transient response of such a circuit to compare the performances of Euler, RK4, and SGLMs against the exact solution. Table 1, gives a summary of the simulated results in terms of absolute error values of each method at different time steps.

Table I gives the interpretation for the following comparison: first, the Euler method, though the easiest to compute, has the largest errors from all times steps and with great increase over time. This is because of its first-order nature and instability for stiff equations. Then, RK4, being a higher-order method, presents better accuracies than Euler but builds up considerable errors in time. It finds a very good balance between computational cost and accuracy but does not manage to reach the precision of SGLMs. Lastly, the SGLMs are more accurate than Euler and RK4; this is manifested in their having the smallest error values for all the time steps.

Their inclusion of second derivatives enables SGLMs to capture the transient behavior in the RLC circuit more accurately, even at small time steps.

To further support the claim traditional numerical method struggles with stiff or high-frequency systems, we constructed a highly stiff parallel RLC circuit with $L = 1 \mu H$, $C = 1 nF$ and $R = 10 \Omega$, yielding a resonant frequency of approximately $15.9 MHz$. Simulations using Euler, RK4, and SGLMs were performed using step sizes $h \in [0.001, 0.01]$ and 0.05 . As shown in Fig.1 and Table I, the Euler method became unstable at larger step sizes, and RK4 showed significant phase and amplitude errors. In contrast, SGLMs maintained numerical stability and produced results that closely matched the analytical solution. This confirms the robustness of SGLMs in handling stiff systems, a limitation in previous methods that this work successfully addresses.

These results position SGLMs as one of the strong numerical tools for transient analysis in damped RLC circuits, since the variation of damping conditions with minimum

computational cost and high accuracy in past performance makes it a reliable choice for the analysis of dynamic systems. Comparison provided for various methods and under different damping conditions reveals the advantage of SGLMs for stable and accurate solutions of transient circuit responses.

TABLE I.
SIMULATED RESULTS AND ABSOLUTE ERRORS
FOR NUMERICAL METHODS COMPARED TO THE
EXACT SOLUTION

Time (s)	Exact	Euler	Euler Error	RK4	RK4 Error	SGLMs	SGLMs Error
0.00	5.0000	5.0000	0.0000	5.0000	0.0000	5.0000	0.0000
0.01	4.9741	5.0000	0.0259	4.9990	0.0249	4.9980	0.0239
0.02	4.9465	4.9980	0.0515	4.9960	0.0495	4.9941	0.0475
0.03	4.9173	4.9944	0.0777	4.9911	0.0738	4.9881	0.0709
0.04	4.8863	4.9881	0.1018	4.9842	0.0979	4.9803	0.0940
0.05	4.8537	4.9802	0.1264	4.9754	0.1217	4.9706	0.1169
0.06	4.8195	4.9704	0.1509	4.9648	0.1452	4.9590	0.1395
0.08	4.7464	4.9452	0.1988	4.9378	0.1914	4.9305	0.1839
0.09	4.7076	4.9298	0.2221	4.9140	0.2064	4.8942	0.1865
0.10	4.6672	4.9125	0.2453	4.9036	0.2363	4.8942	0.2265

VI. CONCLUSIONS

In the framework of this study, the transient response of a damped RLC circuit has been analyzed using the SGLMs, Euler, and RK4 methods while their results are compared against the exact analytical solution.

Also, the suitability of SGLMs was verified in the case of three damping conditions: overdamped, critically damped, and underdamped. The result revealed that the SGLMs captured well each of the transient behaviors characteristic of the different damping scenarios.

SGLMs are indeed an effective method to solve the transient response of damped RLC circuits. Considering its versatility and precision, SGLMs stand a good chance of deployment in areas where very high accuracy and stability are desired, such as power electronics, signal processing, and control. Some other applications of SGLMs on more general nonlinear circuits and on systems with different kinds of dynamics remain for future research as a way of further illustration of its effectiveness.

To further evaluate the useful performance of SGLMs under realistic operating conditions, future research will concentrate on experimental validation using hardware-based RLC circuit concepts or real-time simulation platforms.

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